Math Club 5/9/2011

THEY'RE ALL COMING TO GET YOU

(how mathematicians fool you with their deceptive proofs)

THE MATH GAME

- I will present a completely incorrect proof
- You have to pinpoint the error in the proof

EXAMPLE

- I shall prove that $2 = 1$.
- Let $a = b$.
- $a^2 = ab$
- $2a^2 = a^2 + ab$
- $2a^2 - 2ab = a^2 - ab$
- $2(a^2 - ab) = 1(a^2 - ab)$
- $2 = 1$
- The fallacy is the last step.
- Since $a = b$, we are dividing both sides by zero!

ROUND 1

- I shall prove that $1 = 0$.
- Take the statement $x = 1$.
- Take the derivative of both sides: $\frac{d}{dx}x = \frac{d}{dx}1$
- Then $1 = 0$.
- What went wrong?

ROUND 2

- I shall prove that $1 + 2 + 4 + 8 + 16 + \cdots = -1$.
- Let $x = 1 + 2 + 4 + 8 + \cdots$.
- Then $2x = 2 + 4 + 8 + 16 + \cdots$.
- $2x - x = -1$
- $1 + 2 + 4 + 8 + \cdots = -1$
- What went wrong?

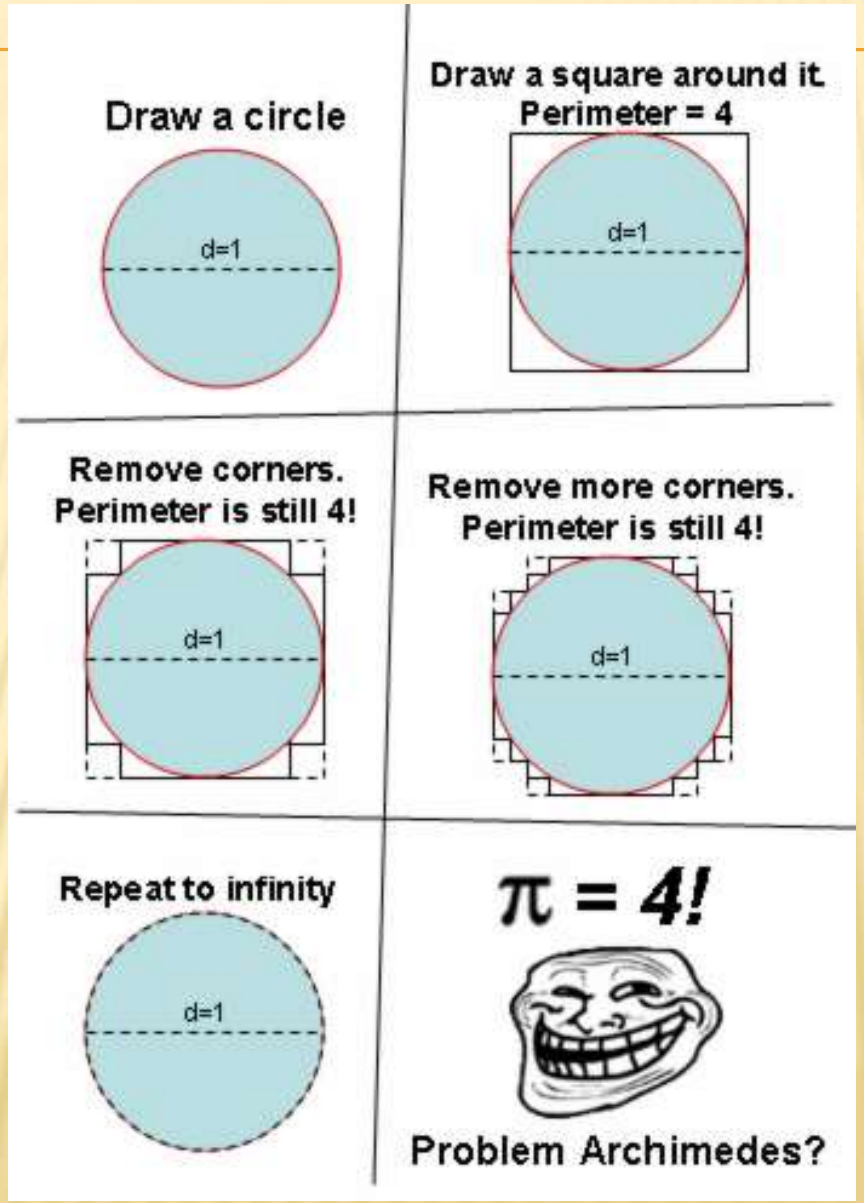
ROUND 3

- I shall prove by induction that in any group of n people, either they are all boys, or they are all girls.
- This is obviously true for $n = 1$.
- Let G be any group of $n + 1$ people. We prove that any two people x and y in G are of the same sex.
- Consider everyone except x. All of them are the same sex.
- Also consider everyone except y. All of them are the same sex
- Take any member z, who has the same sex as x and y.
- Therefore any x and y in the group have the same sex.
- The theorem is proved. What went wrong?

ROUND 4

- I shall prove that $0 = 1$.
- Begin by evaluating $\int \frac{1}{x} dx$ by parts.
- Let $u = \frac{1}{x}$ and $\mathrm{d}v = dx$.
- Then $du = -\frac{1}{x^2} dx$ and $v = x$.
- Hence $\int \frac{1}{x} dx = \frac{x}{x} - \int -\frac{x}{x^2} dx = 1 + \int \frac{1}{x} dx$.
- Therefore $0 = 1$. What went wrong?

ROUND 5


Draw a circle
d=1
Draw a square around it
Perimeter = 4
d=1
Remove corners.
Perimeter is still 4!
d=1
Remove more corners.
Perimeter is still 4!
d=1
Repeat to infinity
d=1
π = 4!
Problem Archimedes?

ROUND 6

- I can destroy the universe with a pencil!
- Assume that there is no friction, pencil is uniform, etc, etc.
- Put the pencil up to a wall and pull one end away with constant velocity while the other end slides down the wall.
- As your end of the pencil is distance x from the wall, the other end is distance $y = \sqrt{L^2 - x^2}$ where L is the length of the pencil.
- Differentiating with respect to time, $\frac{dy}{dt} = -\frac{x\frac{dx}{dt}}{\sqrt{L^2-x^2}}$
- But $\frac{dx}{dt} = v$ so $\frac{dy}{dt} = -\frac{xv}{\sqrt{L^2-x^2}}$
- So $\frac{dy}{dt}$ approaches $-\infty$ as $x \to L$. The pencil crashes on the ground at infinite speed and infinite force!